The ODEs for spherical (3D) double pendulum

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\begin{aligned}
\ddot{\theta}_1 &= \frac{\text{Num}_1}{\text{Denom}^* L_1} \\
\ddot{\theta}_2 &= \frac{\text{Num}_2}{\text{Denom}^* L_2} \\
\ddot{\varphi}_1 &= \frac{\text{Num}_3}{\text{Denom}^* L_1 \sin\theta_1} \\
\ddot{\varphi}_2 &= \frac{\text{Num}_4}{\text{Denom}^* L_2 \sin\theta_2}
\end{aligned}
$$

where

$$\text{Denom} = -(m_2 \sin^2\theta_1 \sin^2\theta_2 \cos^2(\varphi_1-\varphi_2) + \frac{1}{2} m_2 \sin 2\theta_1 \sin 2\theta_2 \cos(\varphi_2-\varphi_1) + m_2 \cos^2\theta_1 \cos^2\theta_2 - m)$$

$$\text{Num}_1 = m_2 \sin\theta_1 \sin^2\theta_2 (L_1 \dot{\theta}_1^2 \cos\theta_1 + g)\cos^2(\varphi_1-\varphi_2) + (-L_2\dot{\varphi}_2^2 \cos\theta_1 \cos^2\theta_2 + (L_1(\dot{\varphi}_1^2 + 2\dot{\theta}_1^2)\cos^2\theta_1 + g\cos\theta_1 - L_1(\dot{\varphi}_1^2 + \dot{\theta}_1^2))\cos\theta_2 + L_2(\dot{\varphi}_2^2 + \dot{\theta}_2^2)\cos\theta_1) m_2 \sin\theta_2 \cos(\varphi_2 - \varphi_1) - \sin\theta_1(-L_2 m_2 \dot{\varphi}_2^2 \cos^3\theta_2 + m_2 L_1(\dot{\varphi}_1^2 + \dot{\theta}_1^2)\cos\theta_1 \cos^2\theta_2 + L_2 m_2(\dot{\varphi}_2^2 + \dot{\theta}_2^2)\cos\theta_2 + m(-L_1\dot{\varphi}_1^2 \cos\theta_1 + g))$$

$$\text{Num}_2 = -(-\frac{1}{2} L_2 m_2 \dot{\theta}_2^2 \sin 2\theta_2 \sin^2\theta_1 \cos^2(\varphi_1-\varphi_2) - \sin\theta_1(-\dot{\varphi}_1^2 L_1 m \cos^2\theta_1 \cos\theta_2 + (L_2 m_2(\dot{\varphi}_2^2 + 2\dot{\theta}_2^2)\cos^2\theta_2 + gm\cos\theta_2 - L_2 m_2(\dot{\varphi}_2^2 + \dot{\theta}_2^2))\cos\theta_1 + L_1 m(\dot{\varphi}_1^2 + \dot{\theta}_1^2)\cos\theta_2)\cos(\varphi_2 - \varphi_1) + (-L_1 m \dot{\varphi}_1^2 \cos^3\theta_1 + (L_2 m_2(\dot{\varphi}_2^2 + \dot{\theta}_2^2)\cos\theta_2 + gm)\cos^2\theta_1 + L_1(\dot{\varphi}_1^2 + \dot{\theta}_1^2) m \cos\theta_1 - L_2 m \dot{\varphi}_2^2 \cos\theta_2)\sin\theta_2)$$

$$\text{Num}_3 = -(m_2(\sin\theta_1 \sin^2\theta_2(-L_1\dot{\varphi}_1^2 \cos^2\theta_1 + g\cos\theta_1 + L_1(\dot{\varphi}_1^2 + \dot{\theta}_1^2))\cos(\varphi_2 - \varphi_1) + \sin\theta_2(-L_1\dot{\varphi}_1^2 \cos^3\theta_1 \cos\theta_2 + g\cos^2\theta_1 \cos\theta_2 + L_1 \cos\theta_1 \cos\theta_2(\dot{\varphi}_1^2 + \dot{\theta}_1^2) - L_2(\dot{\varphi}_2^2 \cos^2\theta_2 - (\dot{\varphi}_2^2 + \dot{\theta}_2^2))))\sin(\varphi_1-\varphi_2) - 2\dot{\theta}_1 L_1(m_2 \sin^2\theta_1 \sin^2\theta_2 \cos^2(\varphi_1-\varphi_2) + \frac{1}{2} m_2 \sin 2\theta_1 \sin 2\theta_2 \cos(\varphi_2 - \varphi_1) + m_2 \cos^2\theta_1 \cos^2\theta_2 - m)\dot{\varphi}_1 \cos\theta_1)$$

$$\text{Num}_4 = (-m_2 L_2 \sin\theta_2(\dot{\varphi}_2^2 \cos^2\theta_2 - (\dot{\varphi}_2^2 + \dot{\theta}_2^2))\sin^2\theta_1 \cos(\varphi_2-\varphi_1) + \sin\theta_1(-L_1\dot{\varphi}_1^2 m \cos^2\theta_1 + (-L_2 m_2 \dot{\varphi}_2^2 \cos^3\theta_2 + L_2 m_2(\dot{\varphi}_2^2 + \dot{\theta}_2^2)\cos\theta_2 + gm)\cos\theta_1 + L_1(\dot{\varphi}_1^2 + \dot{\theta}_1^2) m))\sin(\varphi_1 - \varphi_2) + 2\cos\theta_2(m_2 \sin^2\theta_1 \sin^2\theta_2 \cos^2(\varphi_1-\varphi_2) + \frac{1}{2} m_2 \sin 2\theta_1 \sin 2\theta_2 \cos(\varphi_2-\varphi_1) + m_2 \cos^2\theta_1 \cos^2\theta_2 - m)\dot{\theta}_2 \dot{\varphi}_2 L_2$$

Encoding in the Taylor Center ODE solver:

Denom = - (m2*sin2Th1Th2*cosPhi2m1up2 + 0.5*m2*sin2The1*sin2The2*cosPhi2m1
+ m2*cos2Th1Th2 - m)

Num1 = m2*sinThe1*sinThe2up2*(L1*cosThe1*dThe1up2 + g)*cosPhi2m1up2
+(-L2*cosThe1*dPhi2up2*cosThe2up2 + (L1*(dPhi1up2 + 2*dThe1up2)*cosThe1up2
+ g*cosThe1 - L1*dPh12dTh12)*cosThe2 + L2*cosThe1*dPh22dTh22)*m2*sinThe2*cosPhi2m1
- sinThe1*(-L2*m2*dPhi2up2*cosThe2up3 + m2*L1*cosThe1*dPh12dTh12*cosThe2up2
+ L2*m2*dPh22dTh22*cosThe2 + m*(-L1*dPhi1up2*cosThe1 + g))

Num2 = -(-0.5*sin2The2*dThe2up2*L2*m2*sinThe1up2*cosPhi2m1up2 - sinThe1*(-
cosThe2*dPhi1up2*L1*m*cosThe1up2 + (L2*m2*(dPhi2up2 + 2*dThe2up2)*cosThe2up2
+ g*m*cosThe2 - L2*m2*dPh22dTh22)*cosThe1 + cosThe2*L1*dPh12dTh12*m)*cosPhi2m1
+ (-dPhi1up2*L1*m*cosThe1up3 + (L2*m2*dPh22dTh22*cosThe2 + g*m)*cosThe1up2
+ L1*dPh12dTh12*m*cosThe1 - cosThe2*dPhi2up2*L2*m)*sinThe2)

Num3 = -(m2*(sinThe1*sinThe2up2*(-L1*dPhi1up2*cosThe1up2 + g*cosThe1
+ L1*dPh12dTh12)*cosPhi2m1 + sinThe2*(- L1*cosThe2*dPhi1up2*cosThe1up3
+ g*cosThe1up2*cosThe2 + L1*cosThe1*cosThe2*dPh12dTh12 - L2*(cosThe2up2*dPhi2up2
- dPh22dTh22)))*sinPhi1m2 - 2*dThe1*L1*(m2*sin2Th1Th2*cosPhi2m1up2 +
0.5*m2*sin2The1*sin2The2*cosPhi2m1 + m2*cos2Th1Th2 - m)*dPhi1*cosThe1)

Num4 = (-m2*L2*sinThe2*(cosThe2up2*dPhi2up2 - dPh22dTh22)*sinThe1up2*cosPhi2m1
+ sinThe1*(-L1*dPhi1up2*m*cosThe1up2 + (-L2*m2*dPhi2up2*cosThe2up3
+ L2*m2*dPh22dTh22*cosThe2 + g*m)*cosThe1 + L1*dPh12dTh12*m))*sinPhi1m2
+ 2*cosThe2*(m2*sin2Th1Th2*cosPhi2m1up2 + 0.5*sin2The1*sin2The2*cosPhi2m1*m2
+ cos2Th1Th2*m2 - m)*dThe2*dPhi2*L2