The ODEs obtained as a result

Here are the right-hand sides of the ODEs in a raw form as Maple outputted them. Maple did not recognize trigonometric expressions which should be simplified.

Determinant $\det A$ appearing in the denominator:

$$\det A = -m_1 m_2^2 L_1^4 L_2^4 \sin^2(\theta_1)\sin^2(\theta_2)(m_2(\sin^2(\theta_1)\sin^2(\theta_2)\cos^2(\varphi_2-\varphi_1)+ 2\sin(\theta_1)\sin(\theta_2)\cos(\theta_1)\cos(\theta_2)\cos(\varphi_2-\varphi_1)+\cos^2(\theta_1)\cos^2(\theta_2))-m_1-m_2)$$

The Numerator $\det A_1$:

$$m_1 m_2^2 L_1^3 L_2^4 \sin^2(\theta_2)\sin^2(\theta_1)(m_2 L_1 \sin(\theta_1)\sin^2(\theta_2)(\cos(\theta_1)(\frac{\partial}{\partial t}\theta_1)^2+g)\cos^2(\varphi_2-\varphi_1)+$$
$$(2L_1\cos(\theta_2)(\cos^2(\theta_1)-\frac{1}{2})(\frac{\partial}{\partial t}\theta_1)^2 - L_1\cos(\theta_2)\sin^2(\theta_1)(\frac{\partial}{\partial t}\varphi_1)^2+$$
$$m_2\cos(\theta_1)(L_2(\sin^2(\theta_2)(\frac{\partial}{\partial t}\varphi_2)^2+(\frac{\partial}{\partial t}\theta_2)^2)+$$
$$g\cos(\theta_2)))\sin(\theta_2)\cos(\varphi_2-\varphi_1) - L_1 m_2 \sin(\theta_1)(\cos(\theta_1)\cos^2(\theta_2)(\frac{\partial}{\partial t}\theta_1)^2+$$
$$\cos(\theta_1)\cos^2(\theta_2)-m_1-m_2)(\frac{\partial}{\partial t}\varphi_1)^2+(L_2 m_2\cos(\theta_2)\sin^2(\theta_2))(\frac{\partial}{\partial t}\varphi_2)^2+$$
$$L_2 m_2\cos(\theta_2)(\frac{\partial}{\partial t}\theta_2)^2+g(m_1+m_2)))$$

The Numerator $\det A_2$:

$$-m_1 m_2^2 L_1^4 L_2^3 \sin^2(\theta_1)\sin^2(\theta_2)(-L_2 m_2\cos(\theta_2)\sin(\theta_2)(\frac{\partial}{\partial t}\theta_2)^2\sin^2(\theta_1)\cos^2(\varphi_2-\varphi_1)-$$
$$\sin(\theta_1)(L_1(m_1+m_2)\cos(\theta_2)\sin^2(\theta_1)(\frac{\partial}{\partial t}\varphi_1)^2+2L_2 m_2\cos(\theta_1)(\cos^2(\theta_2)-\frac{1}{2})(\frac{\partial}{\partial t}\theta_2)^2-$$
$$L_2 m_2\cos(\theta_1)\sin^2(\theta_2)(\frac{\partial}{\partial t}\varphi_2)^2+(m_1+m_2)\cos(\theta_2)(L_1(\frac{\partial}{\partial t}\theta_1)^2+g\cos(\theta_1)))\cos(\varphi_2-\varphi_1)+$$
$$(L_1(m_1+m_2)\cos(\theta_1)\sin^2(\theta_1)(\frac{\partial}{\partial t}\varphi_1)^2+L_2 m_2\cos^2(\theta_1)\cos(\theta_2)(\frac{\partial}{\partial t}\theta_2)^2+m_2 L_2\cos(\theta_2)(\cos^2(\theta_1)-$$
$$m_1-m_2)(\frac{\partial}{\partial t}\varphi_2)^2+(m_1+m_2)\cos(\theta_1)(L_1(\frac{\partial}{\partial t}\theta_1)^2+g\cos(\theta_1)))$$

The Numerator $\det A_3$:

$$-m_1 m_2^2 L_1^3 L_2^4 \sin(\theta_1)\sin^2(\theta_2)((L_1\sin^3(\theta_1)(\frac{\partial}{\partial t}\varphi_1)^2 - L_1(\frac{\partial}{\partial t}\theta_1)^2 - g\cos(\theta_1))\sin^2(\theta_2)\cos(\varphi_2-\varphi_1)+$$

$$m_2(L_1\cos(\theta_1)\cos(\theta_2)(\sin^2(\theta_1)(\frac{\partial}{\partial t}\varphi_1)^2+(\frac{\partial}{\partial t}\theta_1)^2)+L_2\sin^2(\theta_2)(\frac{\partial}{\partial t}\varphi_2)^2+g\cos^2(\theta_1)\cos(\theta_2)+$$

$$L_2(\frac{\partial}{\partial t}\theta_2)^2)\sin(\theta_2))\sin(-\varphi_2+\varphi_1) - 2(m_2(\sin^2(\theta_1)\sin^2(\theta_2)\cos^2(\varphi_2-\varphi_1)+$$

$$2\cos(\varphi_2-\varphi_1)\cos(\theta_1)\sin(\theta_2)\sin(\theta_1)\cos(\theta_2)+\cos^2(\theta_1)\cos^2(\theta_2))-m_1-m_2)\cos(\theta_1)(\frac{\partial}{\partial t}\theta_1)L_1(\frac{\partial}{\partial t}\varphi_1))$$

The Numerator $\det A_4$:

$$m_1 m_2^2 L_1^4 L_2^3 \sin^2(\theta_1)\sin(\theta_2)((m_2 L_2\sin(\theta_2)\sin^2(\theta_1)\sin^2(\theta_2)(\frac{\partial}{\partial t}\varphi_2)^2-(\frac{\partial}{\partial t}\theta_2)^2)\cos(\varphi_2-\varphi_1)+$$

$$\sin(\theta_1)(L_2 m_2\cos(\theta_1)\cos(\theta_2)(\sin^2(\theta_2)(\frac{\partial}{\partial t}\varphi_2)^2+(\frac{\partial}{\partial t}\theta_2)^2)-$$

$$(m_1+m_2)(L_1(\sin^2(\theta_1)(\frac{\partial}{\partial t}\varphi_1)^2+(\frac{\partial}{\partial t}\theta_1)^2)+g\cos(\theta_1))))\sin(\varphi_2-\varphi_1)+$$

$$2L_2(\frac{\partial}{\partial t}\theta_2)\cos(\theta_2)(m_2(\sin^2(\theta_1)\sin^2(\theta_2)\cos^2(\varphi_2-\varphi_1)+2\sin(\theta_1)\sin(\theta_2)\cos(\varphi_2-\varphi_1)\cos(\theta_2)\cos(\theta_1)+$$

$$\cos^2(\theta_2)\cos^2(\theta_1)) - m_1 - m_2)(\frac{\partial}{\partial t}\varphi_2))$$