Initial Conditions of New Periodic Solutions of the Three-Body Problem

Ana Hudomal
Institute of Physics, Belgrade University,
Pregrevica 118, Zemun,
P.O.Box 57, 11080 Beograd, Serbia

I. INTRODUCTION

A. Definition of initial conditions

Initial conditions and periods of three-body orbits, where $\dot{x}_1(0), \dot{y}_1(0)$ are the first particle's initial velocities in the x and y directions, respectively, T is the period. The other two particles' initial conditions are specified by these two parameters, as follows, $x_1(0) = -x_2(0) = -1$, $x_3(0) = 0$, $y_1(0) = y_2(0) = y_3(0) = 0$, $\dot{x}_2(0) = \dot{x}_1(0)$, $\dot{x}_3(0) = -2\dot{x}_1(0)$, $\dot{y}_2(0) = \dot{y}_1(0)$, $\dot{y}_3(0) = -2\dot{y}_1(0)$. The Newton's gravity coupling constant and equal masses are taken as $G = m_{1,2,3} = 1$.

B. Equations of motion

In order to simplify the problem, we will first impose some restrictions on our system:

1. Only the non-relativistic three-body problem will be considered. The bodies move in the field of Newtonian gravity.
2. The three bodies are point masses, which means that the sizes of the bodies are negligible compared to the distances between them.
3. We will consider only planar three-body systems, where all bodies move in a fixed plane. The only requirement for this is that none of the bodies has a component of initial velocity perpendicular to the plane defined by their initial positions.

The position of the i-th body is described by its position vector $\mathbf{r}_i = (x_i, y_i)$. Masses are denoted by m_i, and the gravitational constant by G. A planar three-body system has 6 (3x2) degrees-of-freedom; evolution of the system is therefore described by six differential equations of motion. The first two are:

$$\ddot{x}_1(t) = \frac{Gm_2\left(x_2(t) - x_1(t)\right)}{\left[\left(x_1(t) - x_2(t)\right)^2 + \left(y_1(t) - y_2(t)\right)^2\right]^{3/2}} + \frac{Gm_3\left(x_3(t) - x_1(t)\right)}{\left[\left(x_1(t) - x_3(t)\right)^2 + \left(y_1(t) - y_3(t)\right)^2\right]^{3/2}} \tag{1}$$

$$\ddot{y}_1(t) = \frac{Gm_2\left(y_2(t) - y_1(t)\right)}{\left[\left(x_1(t) - x_2(t)\right)^2 + \left(y_1(t) - y_2(t)\right)^2\right]^{3/2}} + \frac{Gm_3\left(y_3(t) - y_1(t)\right)}{\left[\left(x_1(t) - x_3(t)\right)^2 + \left(y_1(t) - y_3(t)\right)^2\right]^{3/2}}, \tag{2}$$

and the other four can be obtained by cyclic permutations $1 \to 2 \to 3 \to 1$. These equations have to be solved numerically. For a more detailed description of the method that I used, see the paper in American Journal of Physics 2014.

The phase space of this system is 12-dimensional; the trajectory in the phase space is a 12-vector function of time t:

$$\mathbf{X}(t) = \left(\mathbf{r}_1(t), \mathbf{r}_2(t), \mathbf{r}_3(t), \dot{\mathbf{r}}_1(t), \dot{\mathbf{r}}_2(t), \dot{\mathbf{r}}_3(t)\right), \tag{3}$$

where $\dot{\mathbf{r}}_i(t)$ is the velocity of the i-th body. The initial conditions are specified by 12 numbers – components of the vector $\mathbf{X}_0 = \mathbf{X}(0)$.

C. The return proximity function

A solution is absolutely periodic if the trajectory in the phase space returns to the initial point after some finite amount of time – period T: $\mathbf{X}(T) = \mathbf{X}_0$. We define the so-called return proximity function as:

$$d(\mathbf{X}_0, T_0) = \min_{t<T_0} \|\mathbf{X}(t) - \mathbf{X}_0\|. \tag{4}$$

where $\|\mathbf{X}(t)\| = \sqrt{\sum_{i=1}^{3} \mathbf{r}_i(t) + \sum_{i=1}^{3} \dot{\mathbf{r}}_i(t)}$ is the Euclidean norm of the 12-vector $\mathbf{X}(t)$. The return proximity function measures the minimal distance to the initial point $\mathbf{X}_0$ reached during the time interval $[0, T_0]$. The condition for absolute periodicity with period $T < T_0$ is now equivalent to $d(\mathbf{X}_0, T_0) = 0$.

A solution is said to be relatively periodic if all relative positions and relative velocities of the three bodies return to their initial values after period T. All absolutely periodic solutions are also relatively periodic, and all relatively periodic solutions are absolutely periodic in some rotating coordinate system.

Various symmetries of the system can be used to simplify the return proximity function, and to make it suitable for searching for relatively periodic solutions. Translational symmetry can be used to set the total momentum to zero; this is done by changing the coordinate system so that the center-of-mass velocity is set to zero. For simplicity, the coordinate origin will be fixed at the center-of-mass.

$$\mathbf{R}_{CM} = \frac{m_1\mathbf{r}_1 + m_2\mathbf{r}_2 + m_3\mathbf{r}_3}{m_1 + m_2 + m_3} = 0 \tag{5}$$

$$\dot{\mathbf{R}}_{CM} = \frac{m_1\dot{\mathbf{r}}_1 + m_2\dot{\mathbf{r}}_2 + m_3\dot{\mathbf{r}}_3}{m_1 + m_2 + m_3} = 0 \tag{6}$$

In this way the phase space dimension, and thus the number of variables in the return proximity function is reduced to eight (by removing two coordinates and two velocities – four constants of motion).

To this point all the equations were written for the general case of bodies with arbitrary masses. From now on we will deal only with the systems of three bodies with equal masses m. The following procedure can be easily modified for different mass ratios.

D. Jacobi coordinates and the shape-sphere

The graphical representation of the three-body system can be simplified with the use of rotational invariance – by changing the coordinates to the Jacobi ones. Jacobi or relative coordinates are defined by two relative coordinate vectors:

$$\boldsymbol{\rho} = \frac{1}{\sqrt{2}}(\mathbf{r}_1 - \mathbf{r}_2), \qquad \boldsymbol{\lambda} = \frac{1}{\sqrt{6}}(\mathbf{r}_1 + \mathbf{r}_2 - 2\mathbf{r}_3). \tag{7}$$

These coordinates can now be used to define a new return proximity function:

$$d(\mathbf{Y}_0, T_0) = \min_{t<T_0} \|\mathbf{Y}(t) - \mathbf{Y}_0\|, \tag{8}$$

where $\mathbf{Y}$ is a 8-vector $\mathbf{Y}(t) = \left(\boldsymbol{\rho}(t), \boldsymbol{\lambda}(t), \dot{\boldsymbol{\rho}}(t), \dot{\boldsymbol{\lambda}}(t)\right)$ and $\mathbf{Y}_0 = \mathbf{Y}(0)$ contains the initial conditions. The zeros of this reduced return-proximity function correspond to absolutely periodic solutions.

Three independent scalar variables can be constructed from Jacobi coordinates: ρ^2, λ^2 and $\boldsymbol{\rho} \cdot \boldsymbol{\lambda}$. The overall size of the orbit is characterized by the hyperradius $R = \sqrt{\rho^2 + \lambda^2}$. These scalar variables are connected to the unit vector with Cartesian components:

$$\hat{\mathbf{n}} = \left(\frac{2\boldsymbol{\rho} \cdot \boldsymbol{\lambda}}{R^2}, \frac{\lambda^2 - \rho^2}{R^2}, \frac{2(\boldsymbol{\rho} \times \boldsymbol{\lambda}) \cdot \mathbf{e}_z}{R^2}\right). \tag{9}$$

Therefore, every configuration of three bodies (shape of the triangle formed by them, independent of size) can be represented by a point on a unit sphere. This sphere is called the shape-sphere.

II. TABLES OF INITIAL CONDITIONS

Table I: Initial velocities, periods and return proximity functions for all new orbits and some of the old ones. Sequences of orbits are divided by horizontal lines.

Label	$\dot{x}_1(0)$	$\dot{y}_1(0)$	T	r.p.f.
I.2.A	0.306 893	0.125 507	6.236	$7.07{\cdot}10^{-7}$
I.2.B	0.392 955	0.097 579	7.004	$1.61{\cdot}10^{-6}$
I.5.A	0.411 293	0.260 755	20.749	$3.43{\cdot}10^{-7}$
I.8.A	0.412 103	0.283 384	34.248	$3.97{\cdot}10^{-7}$
I.9.A	0.402 712	0.210 015	34.712	$9.74{\cdot}10^{-6}$
I.11.A	0.415 252	0.291 346	47.926	$1.11{\cdot}10^{-5}$
I.12.A	0.408 211	0.243 685	48.487	$2.98{\cdot}10^{-7}$
I.13.A	0.399 129	0.184 708	48.667	$2.02{\cdot}10^{-7}$
I.14.A	0.415 169	0.295 341	61.324	$1.30{\cdot}10^{-7}$
I.16.A	0.404 132	0.219 164	62.447	$2.34{\cdot}10^{-7}$
I.17.A	0.397 220	0.169 198	62.627	$4.44{\cdot}10^{-7}$
I.18.A	0.413 537	0.271 006	76.029	$5.03{\cdot}10^{-7}$
I.19.A	0.407 376	0.238 843	76.221	$2.56{\cdot}10^{-7}$
I.20.A	0.401 559	0.202 266	76.401	$2.51{\cdot}10^{-7}$
I.21.A	0.396 058	0.158 601	76.593	$2.84{\cdot}10^{-6}$
I.22.A	0.409 622	0.251 696	89.986	$3.74{\cdot}10^{-7}$
I.23.A	0.404 679	0.222 598	90.181	$6.68{\cdot}10^{-6}$
I.24.A	0.399 807	0.189 807	90.357	$4.22{\cdot}10^{-6}$
I.25.A	0.395 290	0.150 852	90.563	$3.04{\cdot}10^{-6}$
II.4.A	0.080 584	0.588 836	21.271	$5.39{\cdot}10^{-7}$
II.6.A	0.186 238	0.578 714	33.641	$1.88{\cdot}10^{-6}$
II.8.A	0.144 812	0.542 898	38.062	$1.72{\cdot}10^{-6}$
II.11.A	0.184 279	0.587 188	63.535	$1.64{\cdot}10^{-6}$
II.14.A	0.074 735	0.567 937	68.990	$4.50{\cdot}10^{-4}$
II.14.B	0.108 253	0.609 812	82.136	$3.58{\cdot}10^{-5}$
II.14.C	0.074 721	0.567 935	68.989	$5.24{\cdot}10^{-4}$
II.14.D	0.074 720	0.567 935	68.990	$5.88{\cdot}10^{-4}$
II.14.E	0.074 732	0.567 936	68.991	$2.57{\cdot}10^{-4}$
II.15.A	0.049 051	0.590 194	79.153	$3.83{\cdot}10^{-7}$
II.15.B	0.058 685	0.560 793	71.733	$5.12{\cdot}10^{-5}$
II.15.C	0.047 547	0.564 659	72.400	$1.11{\cdot}10^{-4}$
II.15.D	0.179 107	0.572 603	81.600	$2.98{\cdot}10^{-6}$
II.16.A	0.073 903	0.619 865	95.810	$4.58{\cdot}10^{-5}$
II.16.B	0.155 182	0.579 290	86.875	$6.68{\cdot}10^{-6}$
II.17.A	0.061 053	0.609 177	96.873	$3.98{\cdot}10^{-6}$
II.17.B	0.050 367	0.570 341	83.684	$2.51{\cdot}10^{-6}$
II.17.C	0.179 557	0.581 300	95.474	$5.61{\cdot}10^{-4}$
II.17.D	0.169 326	0.580 200	93.995	$6.85{\cdot}10^{-4}$
II.17.E	0.175 847	0.581 000	94.962	$5.42{\cdot}10^{-4}$
II.17.F	0.176 978	0.581 100	95.120	$1.61{\cdot}10^{-4}$
II.17.G	0.174 801	0.580 900	94.814	$5.32{\cdot}10^{-4}$
II.17.H	0.172 120	0.580 600	94.425	$3.37{\cdot}10^{-4}$
II.17.I	0.170 632	0.580 400	94.200	$1.01{\cdot}10^{-4}$
II.19.A	0.149 394	0.550 488	92.890	$9.20{\cdot}10^{-2}$

Table II: Initial velocities, periods and return proximity functions for all new orbits and some of the old ones. Sequences of orbits are divided by horizontal lines.

Label	$\dot{x}_1(0)$	$\dot{y}_1(0)$	T	r.p.f.
III.3.A.α	0.513 938	0.304 736	17.328	$1.75{\cdot}10^{-6}$
III.3.A.β	0.282 699	0.327 209	10.963	$3.56{\cdot}10^{-6}$
III.9.A.α	0.513 150	0.289 437	50.408	$1.14{\cdot}10^{-6}$
III.9.A.β	0.276 237	0.331 714	32.841	$3.70{\cdot}10^{-7}$
III.12.A.α	0.416 822	0.330 333	55.790	$1.32{\cdot}10^{-6}$
III.12.A.β	0.417 343	0.313 100	54.208	$1.49{\cdot}10^{-6}$
III.13.A.α	0.416 444	0.336 397	63.407	$1.97{\cdot}10^{-7}$
III.13.A.β	0.415 819	0.306 804	60.151	$5.14{\cdot}10^{-7}$
III.15.A.α	0.414 396	0.339 223	70.493	$5.46{\cdot}10^{-7}$
III.15.A.β	0.417 701	0.303 455	66.752	$2.11{\cdot}10^{-6}$
III.15.B.α	0.513 063	0.296 863	85.129	$4.10{\cdot}10^{-5}$
III.15.B.β	0.282 036	0.325 643	54.639	$6.27{\cdot}10^{-7}$
III.15.C.α	0.516 228	0.311 409	88.441	$6.50{\cdot}10^{-7}$
III.15.C.β	0.280 396	0.329 229	54.820	$6.05{\cdot}10^{-7}$
III.16.A.α	0.427 659	0.340 300	80.147	$1.59{\cdot}10^{-4}$
III.16.A.β	0.429 098	0.299 359	74.846	$4.26{\cdot}10^{-6}$
III.18.A.α	0.413 720	0.341 698	84.856	$1.05{\cdot}10^{-6}$
III.18.A.β	0.414 643	0.301 216	79.283	$1.42{\cdot}10^{-5}$
III.19.A.β	0.418 091	0.299 900	86.360	$3.75{\cdot}10^{-2}$
III.21.A.β	0.418 259	0.299 482	92.981	$1.05{\cdot}10^{-4}$
IVa.2.A	0.464 445	0.396 060	14.894	$9.92{\cdot}10^{-7}$
IVa.4.A	0.439 166	0.452 968	28.670	$6.22{\cdot}10^{-7}$
IVa.4.B	0.382 604	0.459 000	25.051	$3.18{\cdot}10^{-3}$
IVa.4.C	0.379 619	0.459 500	24.912	$7.44{\cdot}10^{-2}$
IVa.4.D	0.462 608	0.397 137	29.707	$3.64{\cdot}10^{-4}$
IVa.6.A	0.429 090	0.475 313	42.830	$9.75{\cdot}10^{-8}$
IVa.8.A	0.536 917	0.453 083	77.511	$8.04{\cdot}10^{-4}$
IVa.8.B	0.556 677	0.434 231	79.212	$1.27{\cdot}10^{-5}$
IVa.8.C	0.559 188	0.427 252	78.087	$2.42{\cdot}10^{-3}$
IVa.8.D	0.443 173	0.492 707	62.038	$6.32{\cdot}10^{-5}$
IVa.8.E	0.445 695	0.492 500	62.523	$1.53{\cdot}10^{-3}$
IVa.8.F	0.401 574	0.490 047	54.087	$5.62{\cdot}10^{-7}$
IVa.8.G	0.396 528	0.492 290	53.746	$1.95{\cdot}10^{-5}$
IVa.8.H	0.405 043	0.492 281	55.039	$1.34{\cdot}10^{-6}$
IVa.8.I	0.536 369	0.370 817	60.487	$3.26{\cdot}10^{-2}$
IVa.8.J	0.581 298	0.389 502	81.711	$8.10{\cdot}10^{-2}$
IVa.8.K	0.470 929	0.463 385	61.416	$3.06{\cdot}10^{-3}$
IVa.9.A	0.405 276	0.496 190	62.300	$9.93{\cdot}10^{-6}$
IVa.9.B	0.409 717	0.439 222	54.057	$1.21{\cdot}10^{-6}$
IVa.10.A	0.401 289	0.498 542	68.553	$3.13{\cdot}10^{-6}$
IVa.10.B	0.504 456	0.397 379	70.834	$2.50{\cdot}10^{-5}$
IVa.11.A.α	0.395 852	0.503 779	75.242	$6.60{\cdot}10^{-7}$
IVa.11.A.β	0.397 055	0.499 666	74.404	$9.93{\cdot}10^{-6}$
IVa.11.B	0.397 055	0.499 666	74.404	$2.34{\cdot}10^{-5}$
IVa.12.A	0.382 405	0.500 500	77.873	$5.97{\cdot}10^{-2}$
IVa.12.B	0.396 136	0.507 599	82.910	$8.50{\cdot}10^{-6}$
IVa.12.B	0.398 039	0.503 891	82.249	$1.27{\cdot}10^{-4}$
IVa.12.C	0.398 039	0.503 891	82.249	$1.27{\cdot}10^{-4}$
IVa.12.D	0.564 500	0.370 600	98.867	$7.21{\cdot}10^{-2}$
IVa.13.A	0.399 096	0.508 000	90.360	$7.57{\cdot}10^{-2}$
IVa.13.B	0.391 290	0.508 391	88.516	$2.68{\cdot}10^{-5}$
IVa.13.C	0.403 099	0.502 600	89.643	$5.75{\cdot}10^{-2}$
IVa.14.A	0.387 473	0.508 716	94.120	$1.45{\cdot}10^{-6}$
IVa.14.B	0.396 148	0.508 693	96.420	$6.49{\cdot}10^{-3}$
IVa.14.C	0.448 047	0.459 645	96.839	$1.08{\cdot}10^{-5}$
IVa.14.D	0.454 267	0.455 164	97.363	$1.74{\cdot}10^{-4}$
IVa.14.E	0.394 501	0.428 999	94.012	$7.33{\cdot}10^{-3}$
IVa.15.A	0.391 105	0.500 608	98.676	$2.19{\cdot}10^{-5}$
IVa.15.B	0.428 275	0.464 017	98.649	$1.34{\cdot}10^{-6}$
IVa.16.A	0.220 100	0.486 000	75.547	$4.71{\cdot}10^{-2}$

Table III: Initial velocities, periods and return proximity functions for all new orbits and some of the old ones. Sequences of orbits are divided by horizontal lines.

Label	$\dot{x}_1(0)$	$\dot{y}_1(0)$	T	r.p.f.
IVb.3.A	0.405 916	0.230 163	13.866	$1.02{\cdot}10^{-7}$
IVb.6.A	0.414 913	0.274 619	27.664	$2.31{\cdot}10^{-7}$
IVb.7.A	0.398 044	0.176 138	27.823	$3.83{\cdot}10^{-7}$
IVb.9.A	0.414 349	0.288103	41.126	$5.27{\cdot}10^{-6}$
IVb.11.A	0.395 637	0.154 450	41.789	$2.17{\cdot}10^{-7}$
IVb.14.A	0.401 619	0.204 794	55.529	$1.31{\cdot}10^{-5}$
IVb.14.B	0.403 492	0.204 300	55.702	$7.35{\cdot}10^{-5}$
IVb.15.A	0.415 696	0.296 400	68.068	$2.18{\cdot}10^{-6}$
IVb.15.B	0.413 501	0.296 800	67.787	$2.96{\cdot}10^{-4}$
IVb.15.C.α	0.349 985	0.250 800	56.842	$2.71{\cdot}10^{-5}$
IVb.15.C.β	0.454 337	0.228 394	68.038	$4.84{\cdot}10^{-6}$
IVb.15.D	0.395 205	0.142 197	55.820	$5.80{\cdot}10^{-5}$
IVb.15.E	0.393 926	0.142 656	55.708	$5.26{\cdot}10^{-5}$
IVb.15.F	0.394 561	0.142 429	55.762	$2.48{\cdot}10^{-6}$
IVb.15.G	0.314 095	0.244 821	60.422	$5.94{\cdot}10^{-5}$
IVb.15.H	0.473 552	0.210 104	78.868	$3.20{\cdot}10^{-6}$
IVb.17.A	0.475 795	0.160 305	79.998	$8.64{\cdot}10^{-7}$
IVb.18.A	0.399 573	0.188 035	69.512	$5.11{\cdot}10^{-6}$
IVb.18.B	0.321 226	0.204 691	62.088	$1.48{\cdot}10^{-6}$
IVb.19.A	0.393 934	0.134 728	69.740	$9.63{\cdot}10^{-7}$
IVb.20.A	0.408 594	0.245 877	83.106	$3.56{\cdot}10^{-6}$
IVb.21.A	0.403 244	0.213 486	83.292	$1.17{\cdot}10^{-6}$
IVb.22.A	0.299 506	0.292 406	80.369	$6.03{\cdot}10^{-7}$
IVb.22.B	0.302 916	0.197 067	73.206	$8.55{\cdot}10^{-6}$
IVb.23.A	0.410 511	0.256 588	96.866	$2.35{\cdot}10^{-7}$
IVb.23.B	0.450 288	0.099 987	92.180	$8.74{\cdot}10^{-5}$
IVb.23.C	0.393 556	0.129 337	83.720	$7.20{\cdot}10^{-6}$
IVb.23.D	0.308 989	0.157 106	74.826	$8.93{\cdot}10^{-6}$
IVb.24.A	0.350 112	0.079 339	79.476	$7.97{\cdot}10^{-6}$
IVb.25.A	0.401 353	0.200 855	97.246	$1.10{\cdot}10^{-6}$
IVb.26.A	0.396 987	0.167 167	97.432	$1.13{\cdot}10^{-5}$
IVb.27.A	0.393 295	0.125 347	97.702	$3.73{\cdot}10^{-5}$
IVc.5.A	0.383 444	0.377 364	25.840	$4.38{\cdot}10^{-7}$
IVc.8.A	0.519 680	0.353 304	57.994	$1.00{\cdot}10^{-3}$
IVc.8.B	0.519 632	0.353 330	57.988	$5.47{\cdot}10^{-4}$
IVc.8.C	0.302 616	0.357 685	37.177	$6.18{\cdot}10^{-7}$
IVc.12.A.α	0.463 804	0.357 385	68.645	$5.46{\cdot}10^{-5}$
IVc.12.A.β	0.429 325	0.373 739	64.874	$2.67{\cdot}10^{-6}$
IVc.12.B	0.535 402	0.348 429	83.984	$5.09{\cdot}10^{-6}$
IVc.12.C	0.417 326	0.293 300	54.853	$9.81{\cdot}10^{-5}$
IVc.13.A	0.410 016	0.253 400	55.377	$4.99{\cdot}10^{-5}$
IVc.15.A	0.415 696	0.296 400	68.068	$2.19{\cdot}10^{-6}$
IVc.15.B	0.417 304	0.296 100	68.275	$2.39{\cdot}10^{-4}$
IVc.15.C	0.338 374	0.276 396	61.185	$3.12{\cdot}10^{-4}$
IVc.15.D	0.339 476	0.276 300	61.284	$3.71{\cdot}10^{-4}$
IVc.16.A	0.471 671	0.252 351	77.840	$9.71{\cdot}10^{-5}$
IVc.16.B	0.411 718	0.266 600	69.037	$3.95{\cdot}10^{-4}$
IVc.16.C	0.412 798	0.266 401	69.175	$1.94{\cdot}10^{-4}$
IVc.17.A	0.413 909	0.344 900	82.863	$4.50{\cdot}10^{-4}$
IVc.17.B	0.476 139	0.268 743	85.381	$6.86{\cdot}10^{-4}$
IVc.18.A	0.415 114	0.297 900	81.303	$1.95{\cdot}10^{-3}$
IVc.19.A	0.403 994	0.366 295	93.786	$3.82{\cdot}10^{-6}$
IVc.19.B.α	0.406 956	0.354 377	94.775	$6.73{\cdot}10^{-7}$
IVc.19.B.β	0.390 504	0.351 287	90.900	$6.79{\cdot}10^{-7}$
IVc.21.A	0.415 603	0.298 500	94.653	$1.29{\cdot}10^{-3}$
IVc.22.A	0.300 248	0.290 454	80.242	$6.56{\cdot}10^{-7}$
IVc.22.B	0.406 702	0.282 083	95.078	$6.36{\cdot}10^{-7}$

Table IV: Initial velocities, periods and return proximity functions for all new orbits and some of the old ones. Sequences of orbits are divided by horizontal lines.

Label	$\dot{x}_1(0)$	$\dot{y}_1(0)$	T	r.p.f.
V.1.A	0.347 113	0.532 727	6.325	$8.58\cdot10^{-7}$
V.1.B	0.339 393	0.536 191	6.290	$2.73\cdot10^{-6}$
V.4.A.α	0.557 809	0.451 774	39.594	$1.00\cdot10^{-6}$
V.4.A.β	0.181 943	0.514 806	18.141	$5.69\cdot10^{-6}$
V.8.A	0.571 999	0.436 901	80.248	$7.61\cdot10^{-3}$
V.8.B	0.151 718	0.529 800	36.871	$2.66\cdot10^{-3}$
V.12.A	0.158 494	0.537 300	56.942	$1.21\cdot10^{-2}$
V.15.A	0.187 116	0.525 972	70.742	$1.20\cdot10^{-6}$
V.16.A	0.162 366	0.530 500	74.608	$2.93\cdot10^{-3}$
V.20.A	0.158 293	0.527 887	92.178	$7.84\cdot10^{-6}$
VI.2.A	0.464 445	0.396 060	14.894	$9.92\cdot10^{-7}$
VI.6.A	0.559 064	0.349 192	55.502	$5.64\cdot10^{-7}$
VI.6.B	0.558 625	0.351 335	55.714	$1.56\cdot10^{-6}$
VI.6.C	0.558 397	0.353 353	55.960	$6.92\cdot10^{-7}$
VI.12.A	0.559 094	0.355 302	112.84	$1.26\cdot10^{-3}$
VII.4.A	0.537 956	0.341 458	26.918	$1.52\cdot10^{-6}$
VII.6.A	0.442 591	0.423 514	35.833	$1.64\cdot10^{-6}$
VII.7.A.α	0.494 752	0.409 250	47.201	$5.62\cdot10^{-7}$
VII.7.A.β	0.473 674	0.431 289	46.885	$7.04\cdot10^{-5}$
VII.7.B	0.409 495	0.362 823	33.867	$1.97\cdot10^{-6}$
VII.7.C	0.523 394	0.342 113	44.931	$2.76\cdot10^{-4}$
VII.9.A	0.476 366	0.378 935	53.472	$6.23\cdot10^{-5}$
VII.10.A	0.434 075	0.460 697	64.121	$1.16\cdot10^{-3}$
VII.10.B	0.395 999	0.352 935	46.129	$6.08\cdot10^{-4}$
VII.11.A	0.458 038	0.409 375	66.352	$3.94\cdot10^{-7}$
VII.13.A	0.505 939	0.389 401	86.418	$5.75\cdot10^{-7}$
VII.13.B	0.492 544	0.437 327	95.074	$3.65\cdot10^{-6}$
VII.13.C	0.490 505	0.404 421	85.386	$2.55\cdot10^{-5}$
VII.13.D	0.413 880	0.347 796	61.845	$8.00\cdot10^{-6}$
VII.14.A	0.415 170	0.295 341	61.323	$4.85\cdot10^{-6}$
VII.15.A	0.344 750	0.393 045	67.387	$6.78\cdot10^{-7}$
VII.15.B	0.294 621	0.415 210	64.917	$5.09\cdot10^{-6}$
VII.16.A	0.408 991	0.345 713	75.021	$2.95\cdot10^{-6}$
VII.17.A	0.416 066	0.297 150	74.789	$1.12\cdot10^{-5}$
VII.18.A	0.396 743	0.370 881	85.934	$5.19\cdot10^{-7}$
VII.20.A	0.415 757	0.298 190	88.046	$2.77\cdot10^{-4}$
VII.20.B	0.415 752	0.298 191	88.045	$3.50\cdot10^{-4}$
VIII.4.A	0.201 678	0.409 896	21.021	$1.15\cdot10^{-5}$
VIII.8.A	0.301 500	0.441 007	39.612	$1.36\cdot10^{-5}$
VIII.10.A	0.268 073	0.443 797	48.896	$4.99\cdot10^{-2}$
VIII.15.A	0.324 059	0.366 571	61.698	$5.72\cdot10^{-6}$
VIII.15.B	0.288 061	0.437 072	74.393	$3.05\cdot10^{-6}$